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Comparison of Typical Shipment Consolidation Programs:

Structural Results

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Abstract: Shipment consolidation, a commonly used strategy in freight transportation, is the practice of consolidating several small items and then dispatching them on the same vehicle. If applied appropriately, a shipment consolidation program may drive out substantial costs in the logistics supply chain. However, the optimal policy for such a consolidation program in a random setting is not yet fully investigated in the literature. In this paper, we study three alternative systematic shipment consolidation programs: Time Policy, Quantity Policy, and Hybrid Policy. We consider, for one echelon of a logistics supply chain, a freight-arrival process that is Poisson and unit-sized. Accordingly, we analyze these policies and obtain structural results for which one policy is superior to the others, on the basis of total cost per order.
Key Words: Stochastic modeling; Logistics supply chain; Freight transportation; Quantity- based / time-based dispatching

1. INTRODUCTION

Shipment consolidation (SCL) is a logistics strategy that combines two or more orders or shipments so that a larger quantity can be dispatched on the same vehicle. This enables considerable economies of scale, greatly reducing the transportation cost per unit weight. The challenge however is to determine a program/policy for shipping the consolidated load that still gives a good service to the customers whose orders are among the first to be placed. In the literature, several SCL policies have been studied using techniques such as simulation, renewal theory, mathematical programming, and queuing theory. The impact of cost savings and the value added by these policies in the overall logistics supply chain have been increasingly recognized both by practitioners and academicians.

There is a variety of extensions in the analysis and practices of consolidation policies. Hall (Hall, 1987) introduces three types of strategies for consolidation: inventory consolidation, vehicle consolidation, and

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terminal consolidation. Tyan et al. (Tyan JC, Wang F, Du TC, 2003) examine implications of freight consolidation for a company in the context of global supply chains. Ülkü (M.A. Ülkü, 2009) introduces the impact of pricing decisions on shipment consolidation and analyzes various policies for consolidating multiple items in the setting of a logistics supply chain. However, the current literature reveals that there are only a few guidelines for computing optimal consolidation parameters, e.g. (Higginson, Bookbinder, 1995) (Higginson, Bookbinder, 2002) (Çetinkaya, Bookbinder , 2003)

Although any SCL program imposes some administrative costs for planning and management, the benefits are mainly derived from lowered per-unit transportation costs and better transportation operations. However, SCL sometimes may lengthen the shipment cycle and thus adversely affect customer service by delaying order receipts.

An SCL program mostly favors the carrier's pickup, delivery and dock-handling costs. For example, a truckload shipment requires only two stops by the carrier: one for pickup at the origin and one for unloading at the destination. By contrast, small shipments require the carrier to make more stops for pickup and delivery. Moreover, it may not even be economical for the carrier to line-haul some operations in which shipments are so small, and pickup or delivery points so scattered.

An SCL program may also allow for faster and consistent transit times, which in turn would result in reduced inventories (safety or in-transit) without changing customer-service standards. Moreover, with faster transit times, capital is tied up in the consignment for a shorter time; fast deliveries may generate earlier payments and speed cash flow (Masters, 1980).

Transportation and inventory costs are greatly impacted by consolidation strategies within various supply chain configurations. To exemplify, one of many such consolidation configurations is displayed in Figure 1.

In that figure, the loads that are destined to a single point D, say a retailer, are consolidated at a single point O, say a warehouse. As depicted, the composition of the loads may be varying in size (volume, weight) or type. The best consolidation policy depends on various constraints such as management policies and objectives, topology of the logistics network, customer instructions, required transit time, product and transportation characteristics, and the cost parameters included in the model. Also, it is important to consider the type of transportation means, e.g. *private carriage* (one's own truck) or *common carriage* (a public for-hire trucking company).

In practice, typical SCL policies comprise the Time Policy (T-P), Quantity Policy (Q-P), and Hybrid Policy (H-P) which is also known as the "Time-and-Quantity Policy." A T-P dispatches each order at a predetermined shipping date T^*, whether or not it is consolidated. In a Q-P, all orders are held and shipped when a minimum consolidated quantity Q^* is reached. Under the H-P, dispatching occurs upon attaining the earliest of "predetermined shipping date T^*" or "the accumulation of a minimum weight or volume Q^*." Because the Q-P uses a quantity-based dispatch policy, it requires continuous review of the accumulating load. One needs to be cautious in implementing this policy for two reasons: First, the orders might not be easily tracked, or the cost of tracking such orders might overwhelm the benefits of consolidation. Second, because the target consolidated load is itself a random variable, the order cannot be given a delivery-time guarantee. However, the T-P enables us to give a time-guarantee while not requiring the cost of monitoring the arrivals of orders.

As seen in the Figure 2, the time it takes to build up to Q^* orders (optimal dispatch quantity) might be less than T^* (optimal time at which load should be dispatched.) Also, we note that the quantity built up by time T^* might be less than Q^*.

Below, we model the dispatch of a consolidated load consisting of the accumulated orders of a single item. Transport is by private carriage. We analyze a simple but non-trivial consolidation setting where the product is unit-sized, and the arrival process is Poisson. We deliberately study unit-sized orders so that the accumulated quantity at any time is simply the number of orders. In doing so, we enable basic intuitions from structural results, rather than facing computationally and analytically prohibitive situations.

2. THE ANALYSIS AND MODEL FORMULATIONS

The problem at hand can be cast as follows: A retailer is solely supplied by a single vendor for a specific product. The demand for that product follows a Poisson distribution with rate A; thus the inter-demand times follow an exponential distribution with mean interarrival time $1/\mathrm{A}$. Let us define Q to be the random variable representing the total quantity to be consolidated. Orders are accumulated and then dispatched according to a particular release policy. Shipment of orders is done on a first-come, first-served fashion.

Suppose that the order arrival process is independent of demand size distribution. Let H be the holding cost per item per unit time. This cost parameter can also be explained as the *consolidation-penalty cost* of holding an order for a unit of time. Also, let K be the fixed cost to dispatch a vehicle of capacity W orders that are of standard size and weight.

Throughout the formulations and analysis onwards, we assume that the delivery of each load is conducted by one's own truck with a constant cost per shipment. Hence, the consolidation costs only involve the inventory carrying and fixed transportation cost per load.

We now focus on the cost formulations of the SCL policies aforementioned. Let us first consider the Q-P. Its formulation assumes that the dispatcher ships the consolidated load whenever Q or more units are accumulated. (Define $\mathbf{E}$ as the expectation operator.) The expected total cost $\mathrm{E}C(Q)$ of this policy is found as $\mathrm{E}C(Q) = \mathrm{K} + \mathrm{H}Q(Q-1)/2\mathrm{A}$, and the expected cost per order is thus

$$\mathrm{E}\bar{C}(Q) = \mathrm{E}C(Q)/Q = \mathrm{K}/Q + \mathrm{H}(Q-1)/2\mathrm{A}. \tag{1}$$

By treating the cost function as a smooth one, it is easy to show that $\mathrm{E}\bar{C}(Q)$ is convex in Q, i.e. $(d^2\mathrm{E}\bar{C}(Q)/d^2Q) \geq 0$. Let $x \wedge y \stackrel{\text{def}}{=} \min\{x, y\}$. Applying first-order conditions to Eq. (1), and constraining Q by the vehicle capacity W, we find the minimizer of $\mathrm{E}\bar{C}(Q)$ as

$$Q^* = W \wedge \sqrt{2\mathrm{KA}/\mathrm{H}}. \tag{2}$$

We note that in a *manufacturing* setting, Burns et al. [8] consider the inventory holding costs at both origin and destination, and obtain a result similar to Eq. (2). Now, excluding the trivial case when $Q^* = W$, and using Eqs. (1) and (2) the optimal expected cost of the Q-P is derived to be

$$\mathrm{E}\bar{C}(Q^*) = \sqrt{2\mathrm{KA}/\mathrm{H}} - \mathrm{H}/2\mathrm{A}. \tag{3}$$

The quantity in this model setting is integer. Theorem 1 offers the following easy-to-use decision rule for finding Q^*_{int}, the cost-minimizing integral value of Q^*.

Theorem 1. The optimal integral dispatch quantity for the Q-P is

$$Q^*_{int} = \begin{cases} \lfloor Q^* \rfloor \stackrel{\text{def}}{=} Q^*_L, & \text{if } 2\mathrm{KA}/\mathrm{H} \leq Q^*_L Q^*_U \\ \lceil Q^* \rceil \stackrel{\text{def}}{=} Q^*_U, & \text{o.w.} \end{cases} \tag{4}$$

Proof. Note that $Q^*_{int} = Q^*_L$ if $\mathrm{E}\bar{C}(Q^*_L) \leq \mathrm{E}\bar{C}(Q^*_U)$. Expanding this condition, and employing the facts that all the parameters are positive, $Q^*_L > 0$, and $Q^*_U - Q^*_L = 1$, the desired result is obtained. ■

Below we provide a numerical example on the use and simplicity of the Proposition1.

A Numerical Example. Suppose $\mathrm{K} = 10$ \$/order, $\mathrm{H} = 1$ \$/order/day, and $\mathrm{A} = 0.5$ orders/day. Then from (4), we get $Q^* = \sqrt{2(10)(0.5)} = 3.16$, thus $Q^*_L = 3$ and $Q^*_U = 4$. By Theorem 1, since $(3)(4) > 10$, the integer solution for the optimal dispatch quantity is $Q^*_{int} = Q^*_L = 3$ orders. (Note that

$E\bar{C}(3) = 5.33 < E\bar{C}(\sqrt{10}) = 5.32 < E\bar{C}(4) = 5.5.)$ However, now suppose $K = 32$ instead. Then, $Q^* = \sqrt{32} = 5.66$, so $Q_L^* = 5$ and $Q_U^* = 6$. Again by Theorem 1, since $(5)(6) \ngeq 32$, we conlude that $Q_{int}^* = Q_U^* = 6$ orders.

Next, we turn our attention to the formulation of the T-P. Consider that such a policy is applied for a cycle length of *T*. Define X_n to be the time between the $(n-1)^{st}$ and the n^{th} order, with $EX = 1/A < \infty$. Without loss of generality, assume that the first order arrives at time 0. Then $S_n = \sum_{i=1}^{i=n} X_i$ will represent the arrival time of the n^{th} order. Via renewal theory and for Poisson arrivals with rate A, the expected number of renewals in the time interval $[0, T]$, $m(T)$, can be shown to equal AT. Now, let us define $Y(T) = T - S_{m(T)}$ as the age of the last order prior to or at time *T*. Using the fact that $EY(T) = 1/A$, see for example [9], we can derive the expected total cost of the T-P during a consolidation cycle length of T as

$$
\begin{aligned}
EC(T) &= K + EC[HX_1 + 2HX_2 + \cdots + [m(T)-1]HX_{m(T)-1}] + m(T)EY(T), \\
&= K + +Hm(T)EY(T) + HE[X]m(T)[m(T)-1]/2], \\
&= K + (HAT^2)/2 + HT/2.
\end{aligned} \tag{5}
$$

Hence, from Eq. (5), the expected average cost per unit time is obtained as

$$E\bar{C}(T) = EC(T)/m(T) = HT/2 + H/2A + K/AT. \tag{6}$$

Define the maximum holding time (i.e. service level; the maximum time one can hold the first-arriving order until it is dispatched, *possibly* in a consolidated load) by T_{max}. Noting from Eq. (6) that $E\bar{C}(T)$ is convex in *T*, the optimal cycle length of the T-P is derived as

$$T^* = T_{max} \wedge \sqrt{2K/HA}. \tag{7}$$

Discard the trivial case when $T^* = T_{max}$. From Eq. (7), we find the optimal cost value of the T-P simply as

$$E\bar{C}(T^*) = \sqrt{2KA/H} + H/2A. \tag{8}$$

So as to formulate the H-P, let us define its minimal expected cost per order by

$$E\bar{C}(Q^*, T^*) = E\bar{C}(Z(Q^*) \wedge T^*), \tag{9}$$

where $Z(Q^*)$ is the time it takes to accumulate Q^* orders. (We remark that $Z(Q^*)$ is Gamma distributed with mean time $1/A$ and shape Q^*.) The current definition via Eq. (9) enables us to compare all the three policies on the same measure, namely, the expected cost per order. We are now ready to prove

Theorem 2. The Q-P yields a lower cost per order than the T-P and H-P.

Proof. First we will show that Q-P yields a lower cost per order than the T-P. Since $H, A > 0$ and using Eqs. (3) and (8), $E\bar{C}(Q^*) = E\bar{C}(T^*) - H/A < E\bar{C}(T^*)$. Now by Eq. (9), we rewrite

$$E\bar{C}(Q^*, T^*) = \begin{cases} E\bar{C}(Q^*), & \text{if } Z(Q^*) \le T^* \\ E\bar{C}(T^*), & \text{o.w.} \end{cases} \tag{10}$$

Let $\Phi = \Pr\{Z(Q^*) \le T^*\}$, i.e. the probability that quantity policy portion of the H-P will be active before the T-P portion. Now, suppose the H-P gives a lower cost per order than Q-P. Then the following constraint should hold.

$$E\bar{C}(Q^*, T^*) = \Phi\, E\bar{C}(Q^*) + (1-\Phi)E\bar{C}(T^*) = \Phi[E\bar{C}(Q^*) - E\bar{C}(T^*)] < E\bar{C}(Q^*) \tag{11}$$

However, Eq. (11) is satisfied only for $\Phi > 1$, which is contrary to the very definition of a probability measure. Hence, $E\bar{C}(Q^*) \le E\bar{C}(Q^*, T^*)$. This completes the proof. ■

Corollary 1. In terms of cost per order, H-P is superior to T-P.

Proof. For some Φ such that $0 < \Phi < 1$, let the cost difference between Q-P and H-P be Δ. Thus by

Theorem 2, $\Delta = \mathrm{E}\bar{C}(Q^*,T^*) - \mathrm{E}\bar{C}(Q^*) \geq 0$. Via the cost relations of these policies

$$\begin{aligned}\mathrm{E}\bar{C}(T^*) &= [\mathrm{E}\bar{C}(Q^*,T^*) - \Phi\mathrm{E}\bar{C}(Q^*)]/(1-\Phi),\\ &= [\mathrm{E}\bar{C}(Q^*,T^*) - \Phi(\mathrm{E}\bar{C}(Q^*,T^*) - \Delta)]/(1-\Phi),\\ &= \mathrm{E}\bar{C}(Q^*,T^*) + \Phi\Delta/(1-\Phi) \geq \mathrm{E}\bar{C}(Q^*,T^*). \blacksquare\end{aligned}$$

Corollary 2. Costs of typical SCL policies, on the basis of cost per order, follow the ranking

$$\mathrm{E}\bar{C}(Q^*) \leq \mathrm{E}\bar{C}(Q^*,T^*) \leq \mathrm{E}\bar{C}(T^*).$$

Proof. Simply by Theorem 1 and Corollary 1. ■

We now look at the conditions under which the cost difference between any of the those policies is insignificant, and thus the policy-choice is not essential. We offer

Corollary 3. H-P gives the same expected cost as Q-P if $\sqrt{2\mathrm{KA}/\mathrm{H}} \longrightarrow 0$, and gives the same expected cost as T-P if $(2\mathrm{KA}/\mathrm{H})(-1 + \mathrm{H}/\mathrm{A}) \geq 3$.

Proof. We investigate two extreme cases. Consider the case $\Phi = 1$, i.e. the required time to reach the optimal Q-P parameter Q^* is probabilistically always less than that of the T-P. That is $\Phi = \Pr\{Q^*$ or more orders arrive before $T^*\} = \sum_{j=Q^*}^{j=\infty}[(AT^*)^j \exp^{-(AT^*)}/j!] = 1$. Yet, this condition is only satisfied when $AT^* = \sqrt{2\mathrm{KA}/\mathrm{H}} \longrightarrow 0$, which can be interpreted as, "The best policy is to ship each order whenever received," if the fixed dispatch cost relative to the holding cost and the arrival rate are very small.

Now consider the complementary case $\Phi = 0$. This other extreme case implies that $\sum_{j=0}^{Q^*-1}[(AT^*)^j \exp^{-(AT^*)}/j!] = 1 - \Phi = 1$. An exact and explicit expression for this term is not possible. However, we can approximate a Poisson distribution with rate AT^* by a Normal distribution with mean rate AT^* and variance AT^*, when AT^* is "large" (generally when $AT^* > 10$), see [10]. Now, from the characteristics of Normal distribution, $(1-\Phi) \longrightarrow 1$ if $Q^* > 3\sqrt{AT^*} + AT^*$. Employing Eqs. (2) and (7), and using the fact that $\sqrt{x} \geq (1/x)$ for $x \geq 1$, we obtain the condition $(2\mathrm{KA}/\mathrm{H})(-1 + \mathrm{H}/\mathrm{A}) \geq 3$. This completes the proof. ■

3. CONCLUDING REMARKS

In this study, we compared the typical SCL programs on the basis of cost per order. We showed that the Quantity Policy, among its time-based counterparts, yields the most cost-effective solution for the SCL problem with unit-sized demands and Poisson arrivals, when transportation is by private carriage. However, it is important to realize that the constraints for dispatching in real life (e.g. order specific express shipments, batch-arrival processes, non-standard sized orders etc.) greatly complicate the problem.

First, we note that a good consolidation program must be designed with regards to the service level (e.g. transit-time of the order). In that respect, a time policy might prove to be useful, since the time-based policy has a smaller mean consolidation cycle length than that of the quantity-based policy. However, with respect to the demand characteristics and the operating environment, an optimally-determined holding time for the first order arrived can be used in combination with the Hybrid Policy to ensure a particular delivery time guarantee. Second, it should be noted that when transportation is by private carriage, the modeling of the optimal SCL program should incorporate transportation realities such as capacity and various other technological constraints aligned with the service level.

We currently work on the distribution-free version of the problem studied in this paper, both for private carriage and for common carriage with discount economies. The impact of the choice of logistics objectives on the efficiency of the consolidation policies is a challenging research extension.

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FIGURES

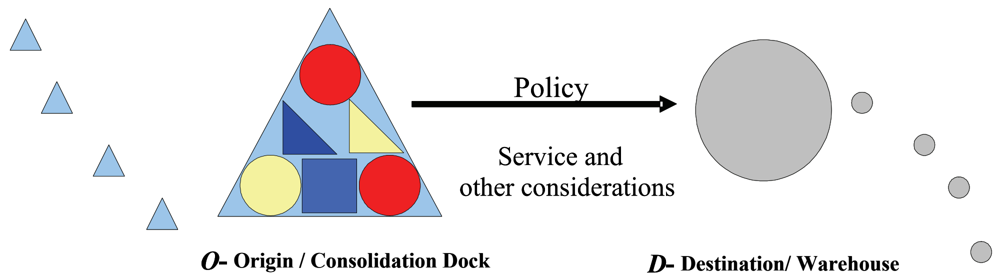

Figure 1: Single Stocking-point, Single-route Consolidation

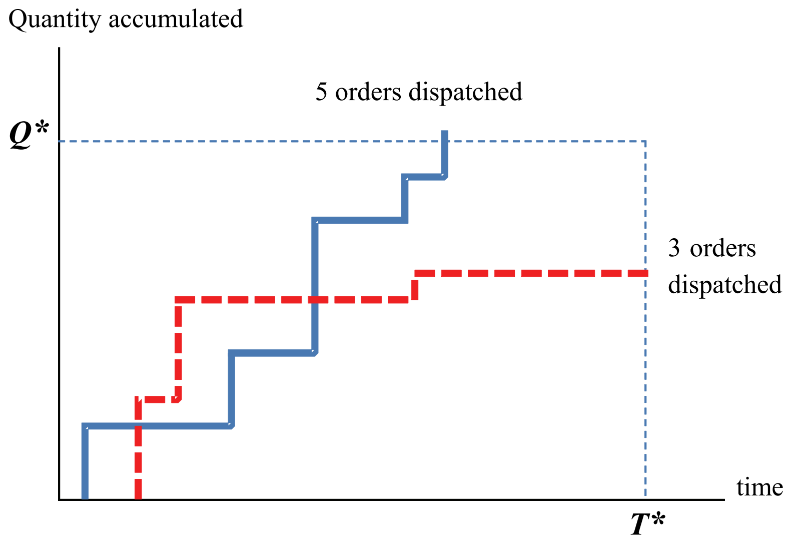

Figure 2: Sample Paths for the Q-P and T-P